Public versus Private Provision of Public Goods

by

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Abstract

It is well known that public goods are underprovided in a static setting with voluntary contributions. Public provision – in a median voter framework with proportional taxation – generally exceeds private provision. This paper compares private and public provision of public goods in a dynamic setting. In a dynamic setting, voluntary donations can result in efficient provision. Also, majority-rule solutions exist even when taxes are not proportional to income. At low discount factors, public provision tends to exceed private provision. As patience increases, however, private provision may exceed public provision. This occurs because many outcomes with a low level of public good provision – and potentially large targeted transfer payments to particular individuals – become sustainable under public provision. Under private provision, however, large targeted transfers are unsustainable. To finance the public good, private provision tends to result in benefit taxation, and public provision tends to result in progressive taxation.

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1. Introduction

It is a well-known theoretical result that a pure public good will be underprovided if its provision is left to private, voluntary donations. There is a large literature on the voluntary provision of public goods (e.g., Warr 1983, Bernheim 1986, Bergstrom et al. 1986, and Andreoni 1988). These papers study a static model of voluntary donations in which individuals simultaneously decide how much to contribute to a pure public good. Underprovision occurs because each individual faces the full marginal cost of providing the public good, yet receives only a fraction of its nonrival benefits.

In contrast, choosing the level of a pure public good through a political process – such as majority rule – can produce higher levels of the public good. Public provision is generally modeled by looking for a Condorcet winner, or a policy that majority-defeats all other policies in pairwise comparison. If the public good is financed by a proportional income tax, a head tax, or any other one-parameter tax system, then the median voter's most preferred level of the public good is the Condorcet winner. Examples of papers using this approach include Barr and Davis (1966), Bowen (1943), Bergstrom (1979), Borcherding and Deacon (1972), Olszewski and Rosenthal (2004), Scotchmer (2002), and Fraser (2003). Epple and Romano (2003) consider a model in which public and private provision coexist, combining the median voter framework with the voluntary contributions game described above. While the median voter's choice of the level of the public good may not be efficient (Bowen 1943, Bergstrom 1979), one does not typically find the severe underprovision that occurs with private provision, as each voter faces a lower marginal cost of providing the public good (the voter's tax share).

While the restriction to one-parameter tax systems is necessary to ensure the existence of a Condorcet winner, it is limiting because it forces a particular kind of income redistribution; for example, under proportional taxation, higher income individuals provide a larger share of the public good consumed equally by all. In order to endogenize the financing of the public good, one must abandon the Condorcet concept and specify a voting game. For example, Baron (1991) specifies a model of legislative bargaining. Lizzeri and Persico (2001) specify a model of Downsian two-party competition and look for a mixed strategy equilibrium (since a pure strategy equilibrium does not exist in the absence of a Condorcet winner). These papers show that allowing income redistribution to be chosen along with the level of the public good creates a new source of inefficiency. In particular, policy makers (e.g., politicians campaigning for office) may prefer to provide targeted benefits in the form of income subsidies rather than diffuse benefits in the form of an increased level of the public good.

These well-known results are derived from static models of both voluntary contributions and majority rule. In this paper, I extend these static models by providing a detailed comparison of private and public provision of public goods in *dynamic* settings. In dynamic models of voluntary donations, cooperation may be sustainable through history-dependent strategies; for instance, non-contributors in the current period may be punished in future periods. Thus, it may be possible to achieve higher – and possibly efficient – levels of the public good. For example, Dickson and Shepsle (2001) show that younger generations in an overlapping generations model can be induced to contribute to a public good through the threat of future punishment. Marx and Matthews (2000) show that efficient outcomes can arise if contributions are made incrementally over time. A similar result holds true for dynamic models of majority rule: allowing history dependence makes many additional outcomes sustainable under majority rule (Bernheim and Slavov 2009), and these additional outcomes may involve lower levels of the public good.

To model private provision in a dynamic setting, I consider an infinitely repeated version of the static voluntary contributions game. To model public provision, I apply Bernheim and Slavov's (2009) notion of a *dynamic Condorcet winner* (DCW), which extends the Condorcet winner concept to dynamic settings. A DCW prescribes a policy for every possible history in such a way that for any history, the prescribed policy choice is majority preferred to any other policy given the implications of the current choice for future outcomes. In contrast to the static setting, a one-parameter tax system is not required to ensure the existence of DCWs. Indeed, DCWs exist with a completely unrestricted tax system, in which each individual pays a different positive or negative tax rate. Lifting the one-parameter restriction on the tax system allows income redistribution to be chosen jointly with the level of the public good.[1] While the DCW concept is intuitively appealing because of its similarity to the static Condorcet concept, applying it in practice can be analytically difficult, even for very simple problems (see, e.g., Bernheim and Slavov 2004, Bernheim and Slavov 2009, Slavov 2006). Thus, an additional contribution of this paper is to demonstrate how DCWs can be found computationally, allowing one to apply it to more complex problems.

I find that for lower discount factors (lower levels of patience), the amount of the public good provided through voluntary donations is often lower than the efficient level. However, as the discount factor rises, higher levels of the public good become sustainable, and efficient provision is attainable. Under public provision, efficient levels of the public good are sustainable even at low discount factors. However, as the discount factor rises, outcomes that involve increased income redistribution, combined with lower levels of the public good, become sustainable. Since a large amount of income redistribution is generally not sustainable under

[1] While other authors have studied dynamic models in which the level of a publicly provided good is determined by majority rule (e.g., Alesina and Rodrick 1994; Bassetto and Benhabib 2006), they generally take a median voter approach in a one parameter tax system.

voluntary donations, even as the discount factor approaches unity, it is unclear whether public provision tends to exceed private provision at higher discount factors. In fact, if all sustainable outcomes are equally likely to arise, then the level of the public good rises (falls) with the discount factor under private (public) provision. Indeed, for high discount factors, private provision results in a higher (closer to efficient) level of the public good than majority rule.

In terms of financing the public good, I find that private provision tends to result in benefit taxation: individuals contribute to the public good in proportion to their taste for it. With income heterogeneity and identical Cobb-Douglas preferences, this typically implies proportional taxation. As the demand for the public good is proportional to income, contributions tend to be proportional to income as well. On the other hand, public provision tends to result in progressive taxation and transfers from the rich to the poor. The intuition for these results is that under private provision, any individual can deviate unilaterally. In this situation, contributions must be roughly proportional to the value individuals place on the public good, as those with a stronger taste for the public good stand to lose more from a punishment in which the level of the public good is lowered. In contrast, under majority rule, unilateral deviations are not allowed. Instead, no majority coalition must have an incentive to deviate from the prescribed policy. Thus, transfers from the rich to the poor are needed in order to ensure that a majority coalition of poorer individuals do not force a switch to a policy that involves a larger contribution from the rich.

This remainder of the paper is organized as follows. Section 2 presents the basic static model of public and private provision; Section 3 describes the dynamic model, solution concepts, and solution method; Section 4 discusses the results; and Section 5 concludes.

2. Static Model

Consider a community of N individuals, $i = 1,...,N$, in which each individual consumes a private good (c_i) and a pure public good (G). To simplify the analysis of majority rule, assume N is odd. Individual i receives an income of Ω_i units of the private good, and aggregate income in the community is $\Omega \equiv \sum_{i=1}^{N} \Omega_i$. One unit of the private good can be transformed into one unit of the public good. Therefore, at the community level, feasibility implies that $\sum_{i=1}^{N} c_i + G = \Omega$. Each individual has a convex utility function $U_i(c_i, G)$, with $\partial U_i / \partial c_i > 0$, $\partial U_i / \partial G > 0$. I assume that $\lim_{c_i \to 0} \partial U_i / \partial c_i = \infty$ and $\lim_{G \to 0} \partial U_i / \partial G = \infty$; that is, as consumption of either good approaches zero, its marginal utility approaches infinity. For each individual i, denote the marginal rate of substitution (MRS) between G and c_i as

$$MRS_i(c_i, G) = \frac{\partial U_i(c_i, G)/\partial G}{\partial U_i(c_i, G)/\partial c_i}.$$

The MRS measures the marginal benefit of the public good to individual i in terms of the private good.

The static solutions to this model – under both public and private provision – are well known, but I describe them here for completeness. Efficient allocations satisfy the Samuelson condition

$$\sum_{i=1}^{N} MRS_i(c_i, G) = 1. \tag{1}$$

That is, due to nonrivalry in consumption, efficiency implies that the sum of the MRSs (the social MRS) of the individuals must equal the marginal cost of the public good in terms of the private good.[2] In general, the efficient level of the public good depends on the distribution of private consumption in the community.

Private provision can be represented by a simultaneous, voluntary contributions game. Each individual simultaneously chooses her contribution to the public good, g_i, taking the others' contributions as given. Individual i's budget constraint is $c_i + g_i = \Omega_i$, and the total amount of the public good is determined by $G = \sum_{i=1}^{N} g_i$. For each individual i, let $G_{-i} = \sum_{j \neq i} g_j$. Individual i's optimal contribution as a function of the others' contributions, $g_i^*(G_{-i})$, satisfies

$$MRS_i\left(\Omega_i - g_i^*\left(G_{-i}\right), G_{-i} + g_i^*\left(G_{-i}\right)\right) = 1 \text{ if } MRS_i\left(\Omega_i, G_{-i}\right) > 1$$

$$g_i^*\left(G_{-i}\right) = 0 \text{ otherwise.}$$

In other words, individual i's best response is to contribute to the public good at a level that equates her MRS to 1, but only if her MRS is greater than 1 at a zero contribution. Otherwise, there is a corner solution, and her best response is a zero contribution. At the Nash equilibrium,

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[2] Strictly speaking, this condition only applies at an interior allocation, in which $G > 0$ and $c_i > 0$ for all i. There may be corner solutions that are efficient but do not satisfy the condition. However, in this particular case, the assumption that $\lim_{G \to 0} \partial U_i / \partial G = \infty$ rules out corner solutions for G, as

$$\lim_{G \to 0} \sum_{i=1}^{N} MRS_i\left(c_i, G\right) = \infty .$$

Moreover, Campbell and Truchon (1988) show that if $MRS_i\left(0, G\right) = 0$ for all i (which is implied by the assumption that $\lim_{c_i \to 0} \partial U_i / \partial c_i = \infty$), condition (1) applies even if $c_i = 0$ for some individuals. Intuitively, suppose $c_i = 0$ for some i and $MRS_i\left(0, G\right) > 0$. Then, although individual i would be willing to sacrifice a positive amount of the private good to increase the level of the public good, she has no private good to sacrifice. Thus, she should receive a smaller weight than the other individuals in the social MRS. However, if $MRS_i\left(0, G\right) = 0$, then such a situation cannot arise, as any individual with zero private good consumption is unwilling to sacrifice any of the private good to increase the level of the public good.

in which each person chooses the best response to the others' contributions, the social MRS is greater than 1, implying underprovision of the public good.[3]

Turning our attention to public provision, suppose that each individual i is charged a tax, $t_i \leq \Omega_i$, to pay for the public good. Thus, $c_i = \Omega_i - t_i$ and, assuming no deadweight loss from taxation, $G = \sum_{i=1}^{N} t_i$. Rather than specify a voting game, I apply the Condorcet winner solution concept: a vector of taxes $(t_1,...,t_N)$ is a Condorcet winner if it is majority-preferred to all other such vectors in pairwise comparison. This is a robust solution concept in the sense that, if a Condorcet winner exists, it emerges in equilibrium across many institutions based on majority rule, including two-party competition (Downs 1957), representative democracy (Besley and Coate 1997), and pairwise voting (Shepsle and Weingast 1984).

If the tax shares are unrestricted, then the degree of income redistribution can be selected independently of the level of the public good. Unfortunately, with such a multidimensional policy set, a Condorcet winner usually fails to exist. While there is no compelling economic reason to restrict attention to one-parameter tax systems, earlier studies have done so in order to guarantee the existence of a majority rule solution. To illustrate this approach, assume that individual i pays for a predetermined share θ_i of the public good, with $\sum_{i=1}^{N} \theta_i = 1$. Thus, the only policy parameter to determine is the level of the public good G, which in turn determines the taxes paid by all individuals. Head taxation occurs if θ_i is the same for all individuals, and proportional income taxation occurs if $\theta_i = \Omega_i / \Omega$. Each individual's utility as a function of G alone is $U_i(\Omega_i - \theta_i G, G)$, and the concavity of U_i ensures that each of these utility functions is

[3] In equilibrium, each contributor to the public good has a marginal rate of substitution that is equal to 1, while each non-contributor has a marginal rate of substitution that is less than 1, but still strictly positive. Thus, the social MRS is strictly greater than 1. If all individuals contribute, then the social MRS is equal to N.

single-peaked in G. The most preferred level of the public good for individual i, $G_i^*(\theta_i)$, is therefore determined by

$$MRS_i\left(\Omega_i - \theta_i G_i^*(\theta_i), G_i^*(\theta_i)\right) = \theta_i \text{ if } MRS_i(\Omega_i, 0) > \theta_i$$

$$G_i^*(\theta_i) = 0 \text{ otherwise.}$$

Clearly, each individual prefers a higher level of the public good compared to the voluntary contributions case. Majority rule results in a level of the public good, G^M, that is equal to the median value of the individuals' $G_i^*(\theta_i)$.

Comparing private and public provision, we can see that it is theoretically ambiguous which one results in higher level of G. However, there are special cases in which it is clear that public provision exceeds private provision. Consider, for example, the case in which individuals have identical preferences – that is, $U_i(c_i, G) = U(c_i, G)$ for all i – and there is proportional income taxation. Then, at the median voter outcome, each individual's MRS is equal to $\theta_i = \Omega_M / \Omega$, where Ω_M is the income of the median voter. The social MRS is $N\,\Omega_M / \Omega$, which is the ratio of the median voter's income to the mean income. The closer the median voter's income is to the mean income in the community, the closer the allocation is to efficiency. If the most preferred level of G is monotonic in income, then Ω_M is also the median income. In this case, the public good is overprovided if the mean income in the community is greater than the median (true for most real-world income distributions). Overprovision occurs because the median voter, whose income is below the mean, uses the public good as a form of income redistribution. To look at another specific example, suppose all individuals have the same income (each earns an equal fraction, Ω/N, of the aggregate income), but different Cobb-Douglas utility functions $U_i(c_i, G) = c_i^{1-\alpha_i} G^{\alpha_i}$. Suppose further that each individual pays $1/N$ of the public good's costs (equivalent to proportional taxation when incomes are identical). It is straightforward to verify that individual i's most preferred level of G is $G_i^* = \alpha_i \Omega$. The median voter is the individual with the median value of α_i – call this α_M. The social MRS at the median voter outcome is

$$\sum_{i=1}^{N} MRS_i = \left(\frac{1-\alpha_M}{\alpha_M}\right)\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\alpha_i}{1-\alpha_i}\right).$$

Thus, if the median taste for the public good (defined by $\alpha_M / (1-\alpha_M)$) is the same as the mean taste for the public good, then the public good is provided at the efficient level. However, with a skewed taste distribution, the public good may be under- or over-provided. In particular, if the benefit from the public good is concentrated among a minority, so that the median taste is smaller than the mean, then the public good will be underprovided.

Intuitively, public provision often results in a higher level of the public good than private provision because each individual only faces a fraction of the cost (the tax share). This principle has been investigated in the literature. For example, Andreoni and Bergstrom (1996) show that the level of the public good can be increased if the government provides a proportional subsidy for voluntary public good purchases and each individual pays a fixed share of government expenditures. This result arises from a similar reduction in the marginal cost of the public good. Bergstrom (1989) and Falkinger (1996) also investigate tax-subsidy schemes that result in the sharing of public good costs.

In addition to cost sharing, repetition may also allow for efficient outcomes, as history-dependent strategies can be used to sustain cooperation. I explore this possibility in the remainder of the paper.

3. Dynamic Model

In the dynamic model, the individuals are infinitely lived and apply a common discount factor $\delta < 1$ to future utility. In every period, $s = 1,2,...$ each individual i earns the same income, Ω_i. To simplify the model, I assume that the public good fully depreciates every period; purchases are not carried into the next period. Thus, the public good in the model is more similar to fireworks or police protection – which are mostly consumed in the period in which they are provided – rather than longer-lasting goods like environmental protection. I also assume that the individuals in the model cannot borrow or save, either individually or collectively through the government. In each period, the level of the public good is determined either by playing the voluntary contributions game described in the previous section, or by majority rule. The key difference in the dynamic setting is that both individual and collective choices can be history dependent.

To characterize the outcomes that are sustainable under majority rule, I use a dynamic analog of the static Condorcet winner concept developed by Bernheim and Slavov (2009). In each period s, the community adopts a vector of taxes t^s using majority rule, with $\sum_{i=1}^{N} t_i^s = G^s$. Let T denote the set of all possible tax vectors, let $h^s = (t^1,...,t^{s-1})$ denote the history of policies adopted through period s, and let H^s denote the set of all feasible histories in period s. A policy program is a mapping, $\mu : \bigcup_{s=1}^{\infty} H^s \rightarrow T$ that specifies for each history, h^s, a policy outcome $\mu(h^s)$. Given any policy program, μ, the continuation path for any history h^s is

$$C^{\mu}(h^s) = \left(\mu(h^s), \mu(h^s, \mu(h^s)), ...\right)$$

A Dynamic Condorcet Winner (DCW) is a policy program μ such that for any history h^s, a majority prefers to implement the prescribed policy $\mu(h^s)$ over any other policy t, given that the resulting continuation paths will be determined by μ. That is, a DCW requires that for any history h^s and any policy $t \in T$,

$$\#\left\{\in (1,\ldots,N) \middle| C^{\mu}\left(h^s\right) \pm_i \left(t, C^{\mu}\left(h^s, t\right)\right)\right\} \geq M \text{ ,}$$

where # denotes the cardinality of the set, $M \equiv (N+1)/2$, and $\pm_i$ denotes the preference relation of individual i over continuation paths. Intuitively, the community considers a one-period, collective deviation to t. In every possible case, such a deviation, followed by the implied punishment, must not be majority preferred to continuing along the prescribed path.

Bernheim and Slavov (2009) show that the DCW concept, like its static counterpart, also summarizes the equilibria of a number of voting institutions. In particular, consider the set of dynamic voting games formed by the infinite repetition of any static voting game that selects Condorcet winners when they exist. This set includes two-party competition, representative democracy (Besley and Coate 1997), and sequential pairwise voting (Shepsle and Weingast 1984). The set of DCW outcomes corresponds to the common equilibrium outcomes of this set of games.

In the dynamic setting, it is no longer necessary for the policy space to be unidimensional, as DCWs generally exist for sufficiently large discount factors. I begin by restricting taxes to be proportional to income (to make the results comparable to a commonly used static model). In this case, $T = \left\{\left(\tau\Omega_1, \tau\Omega_2, \ldots, \tau\Omega_N\right) \mid \tau \in [0,1]\right\}$. Then I relax this restriction and allow each individual's tax to be any mechanically feasible positive or negative value. In this case,

$$T = \left\{\left(t_1, t_2, \ldots, t_N\right) \in \mathbb{R}^N \mid t_i \leq \Omega_i \;\; \forall i, \sum_{i=1}^{N} t_i \geq 0\right\}.$$

Thus, it is possible to use the public budget for targeted transfer payments as well as public good provision. In order to simplify the computational procedure, I restrict attention to stationary policy programs. Stationarity is required both on and off the equilibrium path. Formally, this means that for all h^s, $\mu\left(h^s, \mu(h^s)\right) = \mu\left(h^s\right)$. That is, in the absence of deviation, next period's policy (as well as that of all subsequent periods) will be the same as this period's policy.

In the private provision case, I solve for the set of outcomes that are sustainable in a subgame perfect equilibrium (SPE) of the infinitely repeated voluntary contributions game. To permit a more natural comparison of public and private provision, I allow individuals to make contributions to one another's private good consumption in addition to the public good, effectively combining "private charity" with private provision of the public good. The utility possibilities set under this system is equivalent to that of the public provision model described above, in which each person has an individual-specific positive or negative tax. Clearly, in a static setting, it is never optimal for any player to contribute to another player's private good consumption. However, it may be sustainable in a dynamic setting if failing to provide such a subsidy is punished, for example, by a reduction in others' contributions to the public good. I restrict attention to stationary SPEs. That is, at any point in the game, if there was no deviation in the previous period, all individuals' strategies must specify the same action in the current period as they did in the previous period.

Both the private and public provision models can be solved using a *self-generation mapping*, an algorithm that was developed by Abreu, Pearce, and Stachetti (1990) to find the subgame perfect equilibria of infinitely repeated games, and adapted by Bernheim and Slavov (2009) to find the set of DCWs in dynamic majority rule problems. The self-generation mapping is an iterative procedure that begins with the set of all mechanically feasible average discounted continuation payoffs. Call this set V_0. The stationarity requirement both on and off the equilibrium path implies that V_0 is equal to the utility possibilities set U. This is because any continuation path must consist of repeating the same outcome every period, and the average discounted payoffs from this path can be represented by an element of the utility possibilities set. Starting with this set, one can find the set of payoffs, V_1, that are sustainable given that continuation payoffs must be chosen from V_0. This process is iterated, finding V_2, the set of all payoffs that are sustainable given that continuation payoffs must be chosen from V_1, and so on.

In the public provision case, each set V_k is defined as:

$$V_k = \left\{ v \in V_0 \left| \begin{array}{c} \forall u \in U, \exists w \in V_{k-1} \text{ such that} \\ \#\left\{ i \in (1,\ldots,N) \mid v_i \geq (1-\delta) u_i + \delta w_i \right\} \geq M \end{array} \right. \right\}.$$

That is, a payoff v is in V_k if, for any one-period collective deviation u there exists a "punishment" w in V_{k-1} such that a majority prefers not to deviate. Bernheim and Slavov (2009) demonstrate that when one reaches a fixed point such that $V_k = V_{k-1}$, this set coincides with the set of payoffs sustainable in a DCW.

Note that the particular set of taxes and level of the public good do not matter for the majority-rule solution; only the payoffs to the individuals matter. If a particular set of payoffs is sustainable, then any set of taxes and level of G generating those payoffs is sustainable. This is not the case in the private provision solution. In particular, the optimal deviation for an individual depends on the contributions of the other individuals on the equilibrium path. Modeling this feature requires additional some notation. Recall that in the dynamic setting, individuals have the option to contribute not only to the public good, but also to other individuals' private consumption. Thus, each period, an action (strategy in the stage game) for an individual is a vector $\sigma_i = \left(g_i, f_{i1}, .., f_{i,i-1}, f_{i,i+1}, .., f_{iN}\right) \in \mathbb{R}_+^N$, where g_i is individual i's contribution to the public good and f_{ij} is individual i's transfer payment to individual j. Individual i's action must satisfy $\sum_{j \neq i} f_{ij} + g_i \leq \Omega_i$. Let Δ_i denote the set of feasible actions for individual i, and let $\Delta = \Delta_1 \times ... \times \Delta_N$. Let $\sigma = (\sigma_1, ..., \sigma_N) \in \Delta$ be the profile of actions for all individuals, and let $\sigma_{-i} = (\sigma_1, ..., \sigma_{i-1}, \sigma_{i+1}, ... \sigma_N)$ be the profile of actions for all individuals other than i. Individual i's per-period payoff, as a function of the action profile σ, is given by

$$\eta_i(\sigma) = U_i\left(\Omega_i - \sum_{j \neq i} f_{ij} + \sum_{j \neq i} f_{ji} - g_i, G\right).$$

Let $\eta(\sigma) = \left(\eta_1(\sigma), ..., \eta_N(\sigma)\right)$ be the vector of all individuals' payoffs from action profile σ. Individual i's single-period best response to the others' actions σ_{-i} is to set $f_{ij} = 0$ for all j (transfer payments are never optimal), and to choose her contribution to the public good, $g_i^*(\sigma_{-i})$, to satisfy

$$MRS_i\left(\Omega_i + \sum_{j\,i} f_{ji} - g_i^*(\sigma_{-i}), G_{-i} + g_i^*(\sigma_{-i})\right) = 1 \quad \text{if} \quad MRS_i\left(\Omega_i + \sum_{j\,i} f_{ji}, G_{-i}\right) > 1$$

$$g_i^*(\sigma_{-i}) = 0 \text{ otherwise.}$$

Let $\eta_i^*(\sigma_{-i}) = U_i\left(\Omega_i + \sum_{j\,i} f_{ji} - g_i^*(\sigma_{-i}), G_{-i} + g_i^*(\sigma_{-i})\right)$. In other words, it is the payoff that individual i receives if she chooses her best response to σ_{-i}.

Given this additional notation, the self-generation mapping for the private provision case is defined as:

$$V_k = \left\{ v \in V_0 \middle| \begin{array}{l} \exists \sigma \in \Delta \text{ such that } v = \eta(\sigma) \text{ and} \\ \forall i,\ \eta_i(\sigma_i) \geq \eta_i^*(\sigma_{-i})(1-\delta) + w_i^{'}(V_{k-1})\delta \end{array} \right\}$$

where $w_i^{'}\left(V_{k-1}\right)=\min_{w\in V_{k-1}} w_i$. In other words, a payoff is in V_k if it arises from an action profile with the property that no individual has an incentive to deviate unilaterally (to the static best response), under threat of the worst sustainable punishment in set V_{k-1}. Again, we can find the set of subgame perfect equilibrium payoffs by iterating until $V_k = V_{k-1}$.

Applying the self-generation mapping to specific problems can be difficult if an analytical solution is desired. Slavov (2006) and Bernheim and Slavov (2004, 2009) characterize DCW sets in a number of relatively simple, pure redistribution problems, in which the utility possibilities set is defined by linear constraints. Even in these cases, analytical solutions are difficult to derive. In this paper, I follow a computational approach, which allows me to find solutions to more complex, nonlinear models, such as the one in this paper. There is, however a tradeoff: the computational approach requires me to assume a specific form for the utility functions, and to specify values for the parameters. Nevertheless, I solve the model for a number of different sets of parameter values, attempting to capture some of the interesting features of the problem.

To proceed, I assume that there are three individuals, $i=1,2,3$, and that they each have a Cobb-Douglas utility function, $U_i\left(c_i,G\right)=c_i^{1-\alpha_i}G^{\alpha_i}$.[4] I consider seven sets of parameters, shown in Table I. In each case, total income in the community is $\Omega=2.4$. I discretize the payoff set by requiring individuals to contribute to their income in increments of 0.1. Given the discretization, the self-generation mapping is straightforward to implement. At each iteration, one simply

[4] Using a quasilinear utility function does not substantially alter the conclusions (results available upon request).

cycles through all possible payoffs and checks if they are in the set V_k.[5] (Note that the static best responses must also be chosen from the discretized action sets.)

In the remainder of this paper, for both public and private provision, I define each individual's tax (or contribution) rate, $\tau_i = \left(\Omega_i - c_i\right)/\Omega_i$. Negative tax rates imply that the individual's private good consumption is more than her endowment; that is, she receives net subsidies from the others. Note that these are effective tax rates summarizing the net tax or subsidy for each individual. There may be multiple underlying policies that can give rise to the same net tax rate. For example, consider the case in which all three individuals have $\alpha_i = 0.5$ and $\Omega_i = 0.8$. A situation in which each individual contributes 0.4 units of income to the public good is equivalent in terms of payoffs and net tax rates to one in which individuals 1 and 2 contribute 0.6 each to the public good, and individual 3 provides a transfer payment of 0.2 to each of the first two individuals. In the case of public provision, only the ultimate payoffs matter: as discussed earlier, if the payoffs are sustainable, then any policy generating those payoffs is sustainable. Under private provision, however, it is possible for an outcome to be unsustainable even if it generates the same net tax rates as a sustainable outcome. Returning to the previous example, suppose each individual contributes 0.4 to the public good. Then the optimal deviation (best response) by individual 3 is to set $g_3 = 0$, causing G to fall by 0.4 and individual 3's private good consumption to rise by 0.4. Now suppose individuals 1 and 2 contribute 0.6 each to the public good, and individual 3 provides a transfer payment of 0.2 to

[5] An alternative computational approach is based on Judd et al.'s (2003) method for finding subgame perfect equilibria in infinitely repeated games. This approach relies on convexity of the payoff sets in the self-generation mapping, allowing each set to be represented by its convex hull. In the current problem, the payoff sets can be convexified by allowing public randomization. An earlier version of this paper (available upon request) made this assumption and used the Judd et al. (2003) approach to solve a similar public good contribution problem. The discretization approach used here does not require convexity of the payoff sets.

each of them. The optimal deviation by 3 would still be to set $g_3 = 0$ (and to stop providing transfers to 1 and 2), but the deviation would cause the level G to stay the same and the private good consumption of individuals 1 and 2 to fall by 0.2 each. Because the deviating individual's payoff is different in each case, one outcome may be sustainable while the other is not.

4. Results

To make the comparison between public and private provision, I first provide the static solution to the discretized problem described above. It is useful to note at the outset that because the problem is discrete – forcing reallocations of income to be made in increments of 0.1 – there are efficient allocations that do not result in a social MRS of 1; however, they are close (generally between 0.6 and 1.667). The Nash equilibria for each set of parameter values is characterized in Table II-a. The second through the fourth columns show the effective tax rates for each individual, and the fifth column shows the social MRS (to assess efficiency).[6] In each case, the Nash equilibrium of the continuous model is given in italics; the additional equilibria come from the discretization, which forces deviations to be made in increments of 0.1. Not surprisingly, the social MRS is almost always well above 1, indicating underprovision. The Nash in case 7 – in which individual 3 has most of the income in the community – is efficient given the discretization, but not in the continuous model. Table II-b characterizes the public provision outcomes under the assumption of proportional taxation.[7] The second column indicates the Condorcet winning tax rates (the tax rates favored by the median voter), and the third column indicates the resulting social MRS. In all cases but the third one, the social MRS is

[6] Including the level of the public good itself is not informative, as the efficient level depends on the distribution of private consumption. The social MRS indicates whether the public good is underprovided, overprovided, or efficiently provided.
[7] The public provision solution is for the continuous case. With the restriction to proportional taxation, it is not possible to force all contributions to be made in increments of 0.1 when incomes differ.

considerably lower – and in most cases indicates efficient provision – under majority rule. In case 3, however, preferences vary in an asymmetric way, and the median voter's taste for the public good $(\alpha_2 = 0.25)$ does not reflect the strong preference of a minority $(\alpha_3 = 0.75)$. Under voluntary contributions, however, the individual with the strongest preference can make a large donation $(\tau_3 = 0.75)$, resulting in a more efficient outcome. Thus, public provision is more efficient provided that the benefits from the public good are not concentrated among a minority; if they are, then private provision is more efficient. This result was seen earlier in the more general continuous static model: a skewed distribution of tastes can result in inefficient public provision of the public good. Private provision can also approach efficiency if one individual earns most of the income in the community (case 7).

Now moving to the dynamic setting, but retaining the proportional taxation requirement, it is straightforward to show that the only sustainable outcome under public provision is the static Condorcet winner. To demonstrate this, I first establish that the set of policies $\tau \in [0,1]$ is Condorcet ranked. The proof of this claim is straightforward. Under proportional taxation, $c_i = \Omega_i(1-\tau)$ and $G = \tau\Omega$. Therefore, individual i's indirect utility function over the tax rate is $q_i(\tau) = \Omega_i^{1-\alpha_i}\Omega^{\alpha_i}(1-\tau)^{1-\alpha_i}\tau^{\alpha_i}$. Individual i's most preferred tax rate is $\tau_i^* = \alpha_i$. Without loss of generality, assume that $\alpha_1 \geq \alpha_2 \geq \alpha_3$, making individual 2 the median voter. I will show that the majority preference relation coincides with the preference relation of individual 2. Consider any two tax rates τ and τ', and suppose $q_2(\tau) > q_2(\tau')$. For contradiction, suppose τ is not majority preferred to τ' – that is, $q_1(\tau) < q_1(\tau')$ and $q_3(\tau) < q_3(\tau')$. Thus, we have

(1) $\Omega_1^{1-\alpha_1}\Omega^{\alpha_1}(1-\tau)^{1-\alpha_1}\tau^{\alpha_1} < \Omega_1^{1-\alpha_1}\Omega^{\alpha_1}(1-\tau')^{1-\alpha_1}\tau'^{\alpha_1} \Rightarrow (1-\tau)^{1-\alpha_1}\tau^{\alpha_1} < (1-\tau')^{1-\alpha_1}\tau'^{\alpha_1}$

(2) $\Omega_2^{1-\alpha_2}\Omega^{\alpha_2}\left(1-\tau\right)^{1-\alpha_2}\tau^{\alpha_2} > \Omega_2^{1-\alpha_2}\Omega^{\alpha_2}\left(1-\tau'\right)^{1-\alpha_2}\tau'^{\alpha_2} \Rightarrow \left(1-\tau\right)^{1-\alpha_2}\tau^{\alpha_2} > \left(1-\tau'\right)^{1-\alpha_2}\tau'^{\alpha_2}$

(3) $\Omega_3^{1-\alpha_3}\Omega^{\alpha_3}\left(1-\tau\right)^{1-\alpha_3}\tau^{\alpha_3} < \Omega_3^{1-\alpha_3}\Omega^{\alpha_3}\left(1-\tau'\right)^{1-\alpha_3}\tau'^{\alpha_3} \Rightarrow \left(1-\tau\right)^{1-\alpha_3}\tau^{\alpha_3} < \left(1-\tau'\right)^{1-\alpha_3}\tau'^{\alpha_3}$.

Dividing (1) by (2) implies that

$$\left(\frac{\tau}{1-\tau}\right)^{\alpha_1-\alpha_2} < \left(\frac{\tau'}{1-\tau'}\right)^{\alpha_1-\alpha_2} \Rightarrow \tau < \tau'.$$

Similarly, dividing (2) by (3) implies that $\tau > \tau'$, a contradiction. Thus, τ is majority preferred to τ', and the majority preference relation coincides with the preference relation of individual 2, which is transitive. Bernheim and Slavov (2009) show that if the policy set is Condorcet ranked, the only DCW corresponds to selecting the static Condorcet winner in each period. Thus, the dynamic public provision outcome is no different from the static one characterized in Table II-b.

Under private provision, however, the dynamic solution is quite different from the static one. The sustainable ranges for the social MRS (indicating the level of G relative to efficiency), as well as the sustainable ranges for each of the individuals' tax rates, are shown in Table III-a. The calculation is performed for values of δ ranging from 0.4 (the smallest value for which DCWs exist for all parameter values in Table I) to 0.9, at increments of 0.1, as well as for $\delta = 0.99$. In contrast to the static setting, many levels of G, ranging from underprovision (social MRS substantially greater than 1) to overprovision (social MRS substantially smaller than 1) may be chosen when δ is sufficiently high. Hence the simple comparison from the static setting no longer holds. At sufficiently high discount factors, it is no longer the case that public provision under proportional income taxation results in a higher level of G than private provision. The intuition for this result is a well-known one in game theory, namely that cooperation can be sustained in a dynamic setting through history-dependent strategies. Indeed, as δ approaches unity, a folk theorem result holds, under which any payoff is sustainable provided it gives individuals at least their minmax payoff.

Next, I lift the proportional taxation restriction in the public provision setting, allowing each individual's tax rate to be any feasible positive or negative value. The range of sustainable social MRSs (indicating the level of G relative to the optimum), and the tax rates for each individual, are given in Table III-b. Once proportional taxation is dropped, efficiency is attainable even with asymmetric taste heterogeneity (case 3). Comparing to the private provision results (Table III-a), we can see that under both regimes, higher discount factors result in a larger sustainable set. In general, a greater range of outcomes is sustainable under public provision compared to private provision. For example, with identical individuals and $\delta = 0.8$, the sustainable social MRS ranges from close to zero (severe overprovision) to 23 (severe underprovision). In contrast, under private provision, the social MRS ranges from 0.412 (overprovision, but not as severe) to 7.001 (underprovision, but again, not as severe). The intuition for this result is that there are many outcomes that are vulnerable to unilateral deviations, but not to deviations by a majority; these outcomes are sustainable under public provision but not private provision. In contrast to the static setting, it is not clear that public provision exceeds private provision. For most parameter values, efficient provision (social MRS close to 1) is attainable under either public or private provision, even at lower discount factors. However, with extreme taste heterogeneity (cases 3), efficiency is not attainable under private provision, even for high discount factors; in fact, only the static Nash equilibrium outcome is sustainable. Thus, it appears that income inequality or preference heterogeneity hinder the ability of private provision to achieve efficiency.

There are significant differences between the public and private provision setting in terms of the contributions required to finance the public good. As shown in Tables III-a and III-b, public provision clearly allows for a greater range of net tax rates. A 100 percent tax rate is always sustainable for any individual under public provision, and as δ approaches 1, any one individual can receive almost all of the community's aggregate income for personal consumption of the private good (a tax rate of -175.5 percent is sustainable for an individual with mean income). Under private provision, maximum sustainable contribution rates are always strictly less than 100 percent, and they can be as low as 0 percent for individuals with a low income or weaker taste for the public good. Net subsidies are sustainable, but these tend to be substantially smaller than under public provision. The intuition for this result is that outcomes with very high tax rates on a single individual – particularly on an individual with a lower income or a weaker taste for the public good – are vulnerable to unilateral deviations by the individual facing the high tax rate. However, they are not vulnerable to deviations by a majority provided each of the other two individuals receive a sufficiently large benefit.

The tax rate ranges shown in Tables III-a and III-b include the sustainable tax rates for any level of the public good. They do not make it clear whether the taxes are used to finance the public good or redistribution. To explore this issue further, Tables IV-a and IV-b report the sustainable range of tax rates when $G = 0.4$, the lowest sustainable level with identical individuals. Consider the case of identical individuals (case 1). Such a low level of the public good requires at most a tax rate of 50 percent on a single individual, or a tax rate of 25 percent on two individuals. Once this level of the public good is provided, 2 units of income remain and can be used to provide a subsidy of up to 250 percent to a single individual. Under private provision, it is never possible to exceed a 50 percent tax rate on a single individual, and while private subsidies are possible, they possibility is quite limited, especially for lower values of δ. In contrast, under public provision, a tax rate of 100 percent on any individual is sustainable for any value of δ. Much larger subsidies for private good consumption – up to 137.5 percent – are sustainable as well.

Tables V-a and V-b present the ranges of sustainable tax rates when $G = 1$. When the public good is provided at this relatively high level and tastes vary (cases 2 and 3), lower taxes and larger subsidies are sustainable for individuals who place a smaller marginal value on the private good (individuals 2 and 3). In particular, at any given discount factor, the maximum and minimum tax rates are increasing with taste for the public good (α). One can think of this as a tendency towards benefit taxation. While this effect is present under both public and private provision, it is more pronounced under private provision, with a much narrower range of sustainable tax rates. When incomes vary, public provision requires greater contributions (as a percentage of income) from richer individuals. For example, in case 7 (with extreme income heterogeneity), the highest-income individual pays no less than a 35 percent tax rate when $G = 1$. However, the two low-income individuals can receive large subsidies. One can think of this as a tendency towards progressive taxation. This effect does not appear in a consistent way under private provision.

An alternative way to summarize the multiplicity of outcomes in the dynamic setting is to assume that all sustainable payoffs are equally likely to arise, and compute the expected values of the social MRS, the tax rate for each individual, and the utility levels for each individual. This is done in Figures 1-7 for the seven sets of parameter values described above. In each figure, the left panel shows the expected values under private provision, and the right panel the expected values under public provision.

In terms of the level of the public good, at lower discount factors, public provision tends to result in more efficient provision – and greater utility levels – than private provision. As the discount factor rises, more outcomes become sustainable under both public and private provision. Under private provision, the newly feasible outcomes tend to involve higher levels of the public good. Under public provision, the newly feasible outcomes tend to involve larger transfer payments from some individuals to others, combined with lower levels of the public good. Thus, as the discount factor rises, the social MRS falls and utility levels rise under private provision; the reverse occurs under public provision. Tax rates under public provision fall with the discount factor, but this masks the variation in tax rates across sustainable outcomes. While each individual pays, in expectation, a lower tax rate (reflecting a lower expected level of the public good), there are many outcomes in which some individuals subsidize the private good consumption of others. In Figures 1-6, the social MRS under public provision is lower than that under private provision at lower discount factors (indicating greater efficiency), while the reverse is true at high discount factors. In Figure 7, the social MRS under public provision is roughly equal to that under private provision at lower discount factors, but quickly rises as patience increases.

In terms of distribution, when tastes vary (Figures 2 and 3), private provision tends to result in benefit taxation, while public provision tends to result in roughly equal tax rates. The intuition for this result is that with voluntary contributions, the individual with the weakest taste for the public good can deviate unilaterally and bears a smaller cost than the others from subsequent Nash reversion. This limits the maximum contribution that can be induced from this individual. In contrast, the DCW concept makes it possible for the majority of individuals to impose higher tax rates on the minority with a weaker taste for the public good. When incomes vary (Figures 4-7), public provision tends to result in progressive taxation (with equal utility levels), while private provision tends to result in roughly proportional taxation. The intuition for this result is similar. The possibility of unilateral deviations means that substantial deviations from benefit taxation are unsustainable. With Cobb-Douglas utility, demand for the public good is proportional to income, and benefit taxation implies that individuals contribute in proportion to their income. In contrast, under majority rule, no majority should have an incentive to deviate. Thus, roughly speaking, all majority coalitions must receive a minimum payoff. Because tastes do not vary and income can be redistributed freely, the minimum politically feasible payoff for each coalition is the same. Thus, we are more likely to observe income transfers from the rich to the poor than vice versa, resulting in similar utility levels for all individuals.[8]

Overall, from an efficiency perspective, public provision tends to produce less desirable outcomes than private provision by allowing the possibility of severe underprovision of the public good. On the other hand, if a society dislikes income inequality and prefers progressive taxation, then public provision delivers more desirable outcomes. Thus, in the dynamic setting, there is potentially a tradeoff between equality and efficiency as individuals grow more patient. Public provision results in more equal payoffs for rich and poor through greater redistribution of income, but it risks more severe underprovision of the public good.

One can also interpret the sustainable sets – under either public or private provision – as constraints on policy design. The subgame perfect equilibria of the private provision game represent the outcomes that are sustainable through self-enforcing contracts. That is, they are the outcomes that are achievable when one cannot rely on enforceable contracts or government coercion. A similar interpretation is possible for the DCW set: it is a political feasibility

[8] In a similar vein, Bernheim and Slavov (2004) use the DCW solution concept to study income redistribution and show that majority rule tends to result in transfers to the poor.

constraint requiring a policy choice to be implementable by majority rule (Bernheim and Slavov 2004, Bernheim and Slavov 2009). Self-enforcement feasibility allows a minority to deviate unilaterally. Political feasibility allows a majority to coerce minorities; that is, no individual can deviate unilaterally.

Applying this interpretation, one can imagine a decision maker choosing its preferred outcome subject to either self-enforcement or political feasibility. For example, the decision maker may be a ruling political party with an ideological preference for redistribution towards certain groups. If the public good is provided through the political process, the ruling party must satisfy majoritarian feasibility in order to maintain re-election prospects. If public good provision is "privatized," or if individuals can unilaterally deviate by leaving the jurisdiction, then the ruling party must restrict itself to self-enforcing outcomes. Alternatively, the decision maker may be a dictator who wishes to redistribute resources towards favored groups, but wishes to satisfy a majoritarian or self-enforcement feasibility constraint to avoid the possibility of overthrow. It may also be an interest group that wants to lobby for its preferred outcome (higher payoffs for its members), but recognizes that elected politicians can only choose policies that satisfy majoritarian (or self-enforcement) feasibility.

The results here suggest that as patience increases, it becomes increasingly politically feasible to forgo higher levels of the public good in favor of private redistribution. This kind of redistribution is largely infeasible in a private market, although higher levels of the public good become increasingly feasible as patience increases. The implications of this finding for public good provision depend on the preferences of the individuals. An "interest group" consisting of a single individual would clearly choose to obtain as high a payoff as possible for itself, through either public good or private good consumption. In the current model, Cobb Douglas preferences imply that each individual's demand for the public good rises in proportion to income. While a narrow interest group (a single individual) would redistribute income to its members as much as possible, the level of the public good demanded by the group would increase proportionately. Thus, a narrow interest group would choose a similar level of the public good as a broad coalition, but with a distribution of private good consumption that is skewed towards its members. On the other hand, if demand for the public good did not rise with income (e.g., if individuals had quasilinear utility), then a narrow interest group would choose a lower level of the public good, combined with large income transfers towards its members, than a broad coalition. In this situation, interest groups would, if possible, choose to replace public good provision with targeted transfer payments, and they would have greater opportunity to do so under public provision than under private provision.[9]

A natural question is whether imposing constitutional limits on the size of transfer payments can make the public provision and private provision outcomes more similar to each other. To see whether this is true, I recompute the DCW and SPE sets this time requiring all individuals' net taxes to be nonnegative – that is, net subsidies are not allowed. The computation is done for case 1, in which individuals have identical tastes and incomes. Figure 8 shows the expected values (assuming all sustainable payoffs are equally likely) of the level of the public good, the tax rates, and the utility levels under public and private provision. In addition, the minimum and maximum sustainable tax rates and levels of the public good are given under "Case 8" in Tables III-a, III-b, IV-a, IV-b, V-a, and V-b. The new restriction barely alters the private provision results. This is not surprising given that very large voluntary transfer payments were never sustainable in the first place. Under public provision, however, the social MRS

[9] This intuition is similar to the findings of Lizzeri and Persico (2001) and Baron (1991) discussed earlier.

remains around 1 even as δ rises. Thus, public provision tends to result in greater efficiency than private provision, regardless of the discount factor.

5. Conclusions

This paper has compared the provision of pure public goods by private (voluntary donations) and public (majority rule) means. Most of the previous literature has examined public and private provision of public goods in static settings, with the main finding that public provision financed by proportional taxation and determined by majority rule tends to exceed private provision through voluntary donations. The contribution of this paper has been to show that shifting to a dynamic setting – and allowing history dependence – alters this result. With private provision, I find that it is possible to sustain cooperation and provide the public good efficiently. With public provision, dynamic majority-rule solutions exist even when taxes are not restricted to be proportional to income; thus, income redistribution can be chosen jointly with the level of the public good. At low discount factors, private provision tends to result in less efficient levels of the public good relative to public provision. As patience increases, however, public provision may result in less efficient outcomes than private provision. This possibility arises because larger targeted transfer payments are sustainable under public provision. Such payments become increasingly feasible at higher discount factors, and may result in lower levels of the public good. In terms of financing the public good, private provision tends to result in benefit taxation, with relatively little variation in individual contribution rates. Public provision allows a wider range of tax rates, and there is a tendency towards progressive taxation when incomes vary.

As discussed earlier, finding DCWs analytically is difficult, even in simple cases. While my computational approach has allowed me to consider a more complex model, limits on computing power prevent me from adding many more features to the model. Several useful extensions of this work are possible if one has access to additional computing power, or if one is able to devise a more efficient algorithm. For example, one could consider the implications of deadweight losses from taxation, which would increase the social cost of both income redistribution and provision of the public good. One could also consider the implications of a longer-lasting public good such as environmental protection, or the implications of allowing individuals (or the government) to borrow and save.

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Table I: Parameter Values for Computation

	α1	α2	α3	Ω1	Ω2	Ω3
Case 1	0.5	0.5	0.5	0.8	0.8	0.8
Case 2	0.6	0.5	0.4	0.8	0.8	0.8
Case 3	0.75	0.25	0.25	0.8	0.8	0.8
Case 4	0.5	0.5	0.5	1	0.8	0.6
Case 5	0.5	0.5	0.5	1.2	0.6	0.6
Case 6	0.5	0.5	0.5	1.6	0.8	0
Case 7	0.5	0.5	0.5	2	0.2	0.2

Table II-a: Private Provision in a Static Setting

	Private Provision			
	τ_1	τ_2	τ_3	ΣMRS
Case 1	0.125	0.25	0.375	3
	0.125	0.375	0.25	3
	0.25	0.125	0.375	3
	0.25	***0.25***	***0.25***	***3***
	0.25	0.375	0.125	3
	0.375	0.125	0.25	3
	0.375	0.25	0.125	3
Case 2	***0.5***	***0.25***	***0***	***2.8889***
Case 3	***0.75***	***0***	***0***	***1.8889***
Case 4	0.3	0.25	0.1667	3
	0.3	0.375	0	3
	0.4	0.125	0.1667	3
	0.4	***0.25***	***0***	***3***
	0.5	0.125	0	3
Case 5	0.4167	0	0.1667	3
	0.4167	0.1667	0	3
	0.5	***0***	***0***	***3***
Case 6	0.4375	0.125	a	2
	0.5	***0***	***a***	***2***
Case 7	***0.5***	***0***	***0***	***1.4***

a - Tax rate is undefined because income and contributions are both zero.

Table II-b: Public Provision in a Static Setting

	Public Provision	
	τ (all)	**ΣMRS**
Case 1	0.5	1
Case 2	0.5	1.0556
Case 3	0.25	3.6667
Case 4	0.5	1
Case 5	0.5	1
Case 6	0.5	1
Case 7	0.5	1

Table III-a: Private Provision in a Dynamic Setting

Case 1: Identical Individuals

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	0.625	-0.250	0.625	-0.250	0.625	-0.250	5.001	1.000
0.5	0.625	-0.250	0.625	-0.250	0.625	-0.250	7.001	0.714
0.6	0.625	-0.250	0.625	-0.250	0.625	-0.250	7.001	0.714
0.7	0.750	-0.375	0.750	-0.375	0.750	-0.375	7.001	0.412
0.8	0.750	-0.375	0.750	-0.375	0.750	-0.375	7.001	0.412
0.9	0.750	-0.500	0.750	-0.500	0.750	-0.500	7.001	0.412
0.99	0.875	-0.500	0.875	-0.500	0.875	-0.500	7.001	0.263

Case 2: Taste Heterogeneity

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	0.625	-0.125	0.5	-0.125	0.375	-0.125	5.459	0.986
0.5	0.750	-0.125	0.625	-0.125	0.375	-0.250	5.709	0.667
0.6	0.750	-0.250	0.625	-0.250	0.500	-0.375	5.709	0.578
0.7	0.750	-0.250	0.750	-0.250	0.500	-0.375	5.834	0.479
0.8	0.875	-0.375	0.750	-0.375	0.625	-0.500	7.946	0.306
0.9	0.875	-0.375	0.750	-0.500	0.750	-0.500	8.112	0.254
0.99	0.875	-0.375	0.875	-0.500	0.750	-0.500	8.112	0.192

Case 3: Taste Heterogeneity

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	0.750	0.750	0.000	0.000	0.000	0.000	1.889	1.889
0.5	0.750	0.750	0.000	0.000	0.000	0.000	1.889	1.889
0.6	0.750	0.750	0.000	0.000	0.000	0.000	1.889	1.889
0.7	0.750	0.750	0.000	0.000	0.000	0.000	1.889	1.889
0.8	0.750	0.750	0.000	0.000	0.000	0.000	1.889	1.889
0.9	0.750	0.750	0.000	0.000	0.000	0.000	1.889	1.889
0.99	0.875	0.750	0.125	0.000	0.125	0.000	1.889	0.852

Case 4: Income Heterogeneity

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	0.600	-0.100	0.625	-0.125	0.500	-0.333	5.001	0.846
0.5	0.600	-0.100	0.625	-0.250	0.500	-0.334	5.001	0.714
0.6	0.700	-0.200	0.625	-0.250	0.667	-0.500	5.001	0.600
0.7	0.700	-0.200	0.750	-0.375	0.667	-0.500	7.001	0.412
0.8	0.800	-0.300	0.750	-0.375	0.833	-0.667	7.001	0.263
0.9	0.800	-0.300	0.750	-0.375	0.833	-0.667	7.001	0.263
0.99	0.800	-0.300	0.875	-0.500	0.833	-0.667	7.001	0.263

Case 5: Income Heterogeneity

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	0.583	0.083	0.333	-0.167	0.333	-0.167	3.8	1.182
0.5	0.583	0.083	0.500	-0.167	0.500	-0.167	3.800	0.846
0.6	0.667	-0.083	0.667	-0.333	0.667	-0.333	5.000	0.600
0.7	0.667	-0.083	0.667	-0.500	0.667	-0.500	5.000	0.600
0.8	0.750	-0.083	0.667	-0.500	0.667	-0.500	5.001	0.412
0.9	0.750	-0.250	0.833	-0.667	0.833	-0.667	7.000	0.263
0.99	0.833	-0.250	0.833	-0.667	0.833	-0.667	7.000	0.263

Case 6: Income Heterogeneity

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	0.563	0.312	0.375	0.000	a	a	2.429	1.000
0.5	0.625	0.187	0.500	0.000	a	a	3.000	0.714
0.6	0.625	0.187	0.500	0.000	a	a	3.000	0.714
0.7	0.688	0.063	0.625	0.000	a	a	3.800	0.600
0.8	0.688	0.063	0.750	0.000	a	a	3.800	0.412
0.9	0.688	0.063	0.750	0.000	a	a	3.800	0.412
0.99	0.750	0.063	0.750	0.000	a	a	3.800	0.412

Case 7: Income Heterogeneity

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	0.500	0.500	0.000	0.000	0.000	0.000	1.400	1.400
0.5	0.500	0.500	0.000	0.000	0.000	0.000	1.400	1.400
0.6	0.500	0.500	0.000	0.000	0.000	0.000	1.400	1.400
0.7	0.500	0.500	0.000	0.000	0.000	0.000	1.400	1.400
0.8	0.500	0.500	0.000	0.000	0.000	0.000	1.400	1.400
0.9	0.500	0.500	0.000	0.000	0.000	0.000	1.400	1.400
0.99	0.500	0.500	0.000	0.000	0.000	0.000	1.400	1.400

Case 8: No Transfer Payments

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	0.625	0.000	0.625	0.000	0.625	0.000	5.001	1.000
0.5	0.625	0.000	0.625	0.000	0.625	0.000	7.001	0.714
0.6	0.625	0.000	0.625	0.000	0.625	0.000	7.001	0.714
0.7	0.750	0.000	0.750	0.000	0.750	0.000	7.001	0.412
0.8	0.750	0.000	0.750	0.000	0.750	0.000	7.001	0.412
0.9	0.750	0.000	0.750	0.000	0.750	0.000	7.001	0.412
0.99	0.875	0.000	0.875	0.000	0.875	0.000	7.001	0.200

a - Tax rate is undefined because income and contributions are both zero.

Table III-b: Public Provision in a Dynamic Setting (Unrestricted Taxation)

Case 1: Identical Individuals

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	1.000	-0.250	1.000	-0.250	1.000	-0.250	3.000	0.333
0.5	1.000	-0.500	1.000	-0.500	1.000	-0.500	3.800	0.263
0.6	1.000	-0.750	1.000	-0.750	1.000	-0.750	7.000	0.200
0.7	1.000	-1.125	1.000	-1.125	1.000	-1.125	11.000	0.091
0.8	1.000	-1.375	1.000	-1.375	1.000	-1.375	23.000	0.091
0.9	1.000	-1.625	1.000	-1.625	1.000	-1.625	23.000	0.091
0.99	1.000	-1.750	1.000	-1.750	1.000	-1.750	23.000	0.091

Case 2: Taste Heterogeneity

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	1.000	-0.250	1.000	-0.250	1.000	-0.250	3.306	0.315
0.5	1.000	-0.500	1.000	-0.500	1.000	-0.500	4.800	0.225
0.6	1.000	-0.750	1.000	-0.750	1.000	-0.750	7.722	0.135
0.7	1.000	-1.125	1.000	-1.125	1.000	-1.125	12.333	0.111
0.8	1.000	-1.375	1.000	-1.375	1.000	-1.375	27.000	0.076
0.9	1.000	-1.625	1.000	-1.625	1.000	-1.625	33.500	0.076
0.99	1.000	-1.750	1.000	-1.750	1.000	-1.750	34.000	0.076

Case 3: Taste Heterogeneity

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	1.000	0.000	1.000	-0.250	1.000	-0.250	3.400	0.333
0.5	1.000	-0.375	1.000	-0.500	1.000	-0.500	6.778	0.259
0.6	1.000	-0.625	1.000	-0.750	1.000	-0.750	15.667	0.167
0.7	1.000	-1.000	1.000	-1.125	1.000	-1.125	29.000	0.111
0.8	1.000	-1.250	1.000	-1.375	1.000	-1.375	42.333	0.067
0.9	1.000	-1.500	1.000	-1.625	1.000	-1.625	61.000	0.030
0.99	1.000	-1.750	1.000	-1.750	1.000	-1.750	66.334	0.030

Case 4: Income Heterogeneity

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	1.000	0.000	1.000	-0.250	1.000	-0.667	3.000	0.333
0.5	1.000	-0.200	1.000	-0.500	1.000	-1.000	3.800	0.263
0.6	1.000	-0.400	1.000	-0.750	1.000	-1.333	7.000	0.200
0.7	1.000	-0.700	1.000	-1.125	1.000	-1.833	11.000	0.091
0.8	1.000	-0.900	1.000	-1.375	1.000	-2.167	23.000	0.091
0.9	1.000	-1.100	1.000	-1.625	1.000	-2.500	23.000	0.091
0.99	1.000	-1.200	1.000	-1.750	1.000	-2.667	23.000	0.091

Case 5: Income Heterogeneity

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	1.000	0.167	1.000	-0.667	1.000	-0.667	3.000	0.333
0.5	1.000	0.000	1.000	-1.000	1.000	-1.000	3.800	0.263
0.6	1.000	-0.167	1.000	-1.333	1.000	-1.333	7.000	0.200
0.7	1.000	-0.417	1.000	-1.833	1.000	-1.833	11.000	0.091
0.8	1.000	-0.583	1.000	-2.167	1.000	-2.167	23.000	0.091
0.9	1.000	-0.750	1.000	-2.500	1.000	-2.500	23.000	0.091
0.99	1.000	-0.833	1.000	-2.667	1.000	-2.667	23.000	0.091

Case 6: Income Heterogeneity

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	1.000	0.375	1.000	-0.250	a	a	3.000	0.333
0.5	1.000	0.250	1.000	-0.500	a	a	3.800	0.263
0.6	1.000	0.125	1.000	-0.750	a	a	7.000	0.200
0.7	1.000	-0.063	1.000	-1.125	a	a	11.000	0.091
0.8	1.000	-0.187	1.000	-1.375	a	a	23.000	0.091
0.9	1.000	-0.313	1.000	-1.625	a	a	23.000	0.091
0.99	1.000	-0.375	1.000	-1.750	a	a	23.000	0.091

Case 7: Income Heterogeneity

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	1.000	0.500	1.000	-4.000	1.000	-4.000	3.000	0.333
0.5	1.000	0.400	1.000	-5.000	1.000	-5.000	3.800	0.263
0.6	1.000	0.300	1.000	-6.000	1.000	-6.000	7.000	0.200
0.7	1.000	0.150	1.000	-7.500	1.000	-7.500	11.000	0.091
0.8	1.000	0.050	1.000	-8.500	1.000	-8.500	23.000	0.091
0.9	1.000	-0.050	1.000	-9.500	1.000	-9.500	23.000	0.091
0.99	1.000	-0.100	1.000	-10.000	1.000	-10.000	23.000	0.091

Case 8: No Transfer Payments

δ	1 max	1 min	2 max	2 min	3 max	3 min	ΣMRS max	ΣMRS min
0.4	1.000	0.000	1.000	0.000	1.000	0.000	2.429	0.333
0.5	1.000	0.000	1.000	0.000	1.000	0.000	3.800	0.263
0.6	1.000	0.000	1.000	0.000	1.000	0.000	5.000	0.200
0.7	1.000	0.000	1.000	0.000	1.000	0.000	5.000	0.143
0.8	1.000	0.000	1.000	0.000	1.000	0.000	7.000	0.143
0.9	1.000	0.000	1.000	0.000	1.000	0.000	7.000	0.091
0.99	1.000	0.000	1.000	0.000	1.000	0.000	11.000	0.091

a - Tax rate is undefined because income and contributions are both zero.

Table IV-a: Private Provision in a Dynamic Setting when G=0.4

Case 1: Identical Individuals						
	1		2		3	
δ	max	min	max	min	max	min
0.4	0.500	-0.125	0.500	-0.125	0.500	-0.125
0.5	0.500	-0.125	0.500	-0.125	0.500	-0.125
0.6	0.500	-0.250	0.500	-0.250	0.500	-0.250
0.7	0.500	-0.250	0.500	-0.250	0.500	-0.250
0.8	0.500	-0.375	0.500	-0.375	0.500	-0.375
0.9	0.500	-0.375	0.500	-0.375	0.500	-0.375
0.99	0.500	-0.375	0.500	-0.375	0.500	-0.375

Case 2: Taste Heterogeneity						
	1		2		3	
δ	max	min	max	min	max	min
0.4	0.250	0.125	0.375	0.000	0.375	0.000
0.5	0.375	-0.125	0.500	-0.125	0.375	-0.125
0.6	0.375	-0.125	0.500	-0.125	0.375	-0.125
0.7	0.375	-0.250	0.500	-0.125	0.375	-0.250
0.8	0.375	-0.375	0.500	-0.250	0.375	-0.250
0.9	0.375	-0.375	0.500	-0.250	0.375	-0.375
0.99	0.375	-0.375	0.500	-0.250	0.375	-0.375

Case 3: Taste Heterogeneity						
	1		2		3	
δ	max	min	max	min	max	min
0.4	b	b	b	b	b	b
0.5	b	b	b	b	b	b
0.6	b	b	b	b	b	b
0.7	b	b	b	b	b	b
0.8	b	b	b	b	b	b
0.9	b	b	b	b	b	b
0.99	b	b	b	b	b	b

Case 4: Income Heterogeneity						
	1		2		3	
δ	max	min	max	min	max	min
0.4	0.300	-0.100	0.500	0.000	0.500	-0.167
0.5	0.300	-0.100	0.500	-0.125	0.500	-0.167
0.6	0.300	-0.100	0.500	-0.125	0.500	-0.167
0.7	0.300	-0.200	0.500	-0.250	0.500	-0.333
0.8	0.300	-0.300	0.500	-0.250	0.500	-0.333
0.9	0.300	-0.300	0.500	-0.250	0.500	-0.500
0.99	0.300	-0.300	0.500	-0.250	0.500	-0.500

Case 5: Income Heterogeneity						
	1		2		3	
δ	max	min	max	min	max	min
0.4	b	b	b	b	b	b
0.5	b	b	b	b	b	b
0.6	0.083	-0.083	0.500	0.000	0.500	0.000
0.7	0.167	-0.083	0.500	0.000	0.500	0.000
0.8	0.167	-0.083	0.500	-0.167	0.500	-0.167
0.9	0.167	-0.167	0.500	-0.167	0.500	-0.167
0.99	0.167	-0.167	0.500	-0.167	0.500	-0.167

Case 6: Income Heterogeneity						
	1		2		3	
δ	max	min	max	min	max	min
0.4	b	b	b	b	b	b
0.5	b	b	b	b	b	b
0.6	b	b	b	b	b	b
0.7	b	b	b	b	b	b
0.8	b	b	b	b	b	b
0.9	b	b	b	b	b	b
0.99	b	b	b	b	b	b

Case 7: Income Heterogeneity						
	1		2		3	
δ	max	min	max	min	max	min
0.4	b	b	b	b	b	b
0.5	b	b	b	b	b	b
0.6	b	b	b	b	b	b
0.7	b	b	b	b	b	b
0.8	b	b	b	b	b	b
0.9	b	b	b	b	b	b
0.99	b	b	b	b	b	b

Case 8: No Transfer Payments						
	1		2		3	
δ	max	min	max	min	max	min
0.4	0.500	0.000	0.500	0.000	0.500	0.000
0.5	0.500	0.000	0.500	0.000	0.500	0.000
0.6	0.500	0.000	0.500	0.000	0.500	0.000
0.7	0.500	0.000	0.500	0.000	0.500	0.000
0.8	0.500	0.000	0.500	0.000	0.500	0.000
0.9	0.500	0.000	0.500	0.000	0.500	0.000
0.99	0.500	0.000	0.500	0.000	0.500	0.000

a - Tax rate is undefined because income and contributions are both zero.
b - G=0.4 is not sustainable.

Table IV-b: Public Provision in a Dynamic Setting when G=0.4

Case 1: Identical Individuals

	1		2		3	
δ	max	min	max	min	max	min
0.4	b	b	b	b	b	b
0.5	b	b	b	b	b	b
0.6	1.000	-0.750	1.000	-0.750	1.000	-0.750
0.7	1.000	-1.125	1.000	-1.125	1.000	-1.125
0.8	1.000	-1.250	1.000	-1.250	1.000	-1.250
0.9	1.000	-1.375	1.000	-1.375	1.000	-1.375
0.99	1.000	-1.375	1.000	-1.375	1.000	-1.375

Case 5: Income Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	b	b	b	b	b	b
0.5	b	b	b	b	b	b
0.6	1.000	-0.167	1.000	-1.333	1.000	-1.333
0.7	1.000	-0.417	1.000	-1.833	1.000	-1.833
0.8	1.000	-0.500	1.000	-2.000	1.000	-2.000
0.9	1.000	-0.583	1.000	-2.167	1.000	-2.167
0.99	1.000	-0.583	1.000	-2.167	1.000	-2.167

Case 2: Taste Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	b	b	b	b	b	b
0.5	1.000	0.625	0.125	-0.500	0.125	-0.375
0.6	1.000	-0.750	1.000	-0.750	1.000	-0.750
0.7	1.000	-1.125	1.000	-1.125	1.000	-1.125
0.8	1.000	-1.250	1.000	-1.250	1.000	-1.250
0.9	1.000	-1.375	1.000	-1.375	1.000	-1.375
0.99	1.000	-1.375	1.000	-1.375	1.000	-1.375

Case 6: Income Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	b	b	b	b	b	b
0.5	b	b	b	b	b	b
0.6	1.000	0.125	1.000	-0.750	a	a
0.7	1.000	-0.062	1.000	-1.125	a	a
0.8	1.000	-0.125	1.000	-1.250	a	a
0.9	1.000	-0.188	1.000	-1.375	a	a
0.99	1.000	-0.188	1.000	-1.375	a	a

Case 3: Taste Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	1.000	0.750	-0.125	-0.250	-0.125	-0.250
0.5	1.000	0.250	0.375	-0.500	0.375	-0.500
0.6	1.000	-0.500	1.000	-0.750	1.000	-0.750
0.7	1.000	-1.000	1.000	-1.000	1.000	-1.000
0.8	1.000	-1.125	1.000	-1.375	1.000	-1.375
0.9	1.000	-1.375	1.000	-1.375	1.000	-1.375
0.99	1.000	-1.375	1.000	-1.375	1.000	-1.375

Case 7: Income Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	b	b	b	b	b	b
0.5	b	b	b	b	b	b
0.6	1.000	0.300	1.000	-6.000	1.000	-6.000
0.7	1.000	0.150	1.000	-7.500	1.000	-7.500
0.8	1.000	0.100	1.000	-8.000	1.000	-8.000
0.9	1.000	0.050	1.000	-8.500	1.000	-8.500
0.99	1.000	0.050	1.000	-8.500	1.000	-8.500

Case 4: Income Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	b	b	b	b	b	b
0.5	b	b	b	b	b	b
0.6	1.000	-0.400	1.000	-0.750	1.000	-1.333
0.7	1.000	-0.700	1.000	-1.125	1.000	-1.833
0.8	1.000	-0.800	1.000	-1.250	1.000	-2.000
0.9	1.000	-0.900	1.000	-1.375	1.000	-2.167
0.99	1.000	-0.900	1.000	-1.375	1.000	-2.167

Case 8: No Transfer Payments

	1		2		3	
δ	max	min	max	min	max	min
0.4	b	b	b	b	b	b
0.5	b	b	b	b	b	b
0.6	0.500	0.000	0.500	0.000	0.500	0.000
0.7	0.500	0.000	0.500	0.000	0.500	0.000
0.8	0.500	0.000	0.500	0.000	0.500	0.000
0.9	0.500	0.000	0.500	0.000	0.500	0.000
0.99	0.500	0.000	0.500	0.000	0.500	0.000

a - Tax rate is undefined because income and contributions are both zero.

b - G=0.4 is not sustainable.

Table V-a: Private Provision in a Dynamic Setting when G=1

Case 1: Identical Individuals

	1		2		3	
δ	max	min	max	min	max	min
0.4	0.500	0.250	0.500	0.250	0.500	0.250
0.5	0.625	0.000	0.625	0.000	0.625	0.000
0.6	0.625	0.000	0.625	0.000	0.625	0.000
0.7	0.625	0.000	0.625	0.000	0.625	0.000
0.8	0.750	-0.125	0.750	-0.125	0.750	-0.125
0.9	0.750	-0.250	0.750	-0.250	0.750	-0.250
0.99	0.750	-0.250	0.750	-0.250	0.750	-0.250

Case 2: Taste Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	0.625	0.375	0.5	0.25	0.375	0.125
0.5	0.750	0.250	0.625	0.125	0.375	0.000
0.6	0.750	0.125	0.625	0.000	0.500	-0.125
0.7	0.750	0.125	0.625	0.000	0.500	-0.125
0.8	0.750	-0.125	0.750	-0.125	0.625	-0.250
0.9	0.750	-0.125	0.750	-0.125	0.625	-0.250
0.99	0.750	-0.125	0.750	-0.125	0.625	-0.250

Case 3: Taste Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	b	b	b	b	b	b
0.5	b	b	b	b	b	b
0.6	b	b	b	b	b	b
0.7	b	b	b	b	b	b
0.8	b	b	b	b	b	b
0.9	b	b	b	b	b	b
0.99	b	b	b	b	b	b

Case 4: Income Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	0.600	0.300	0.500	0.125	0.500	0.000
0.5	0.600	0.200	0.625	0.125	0.500	-0.167
0.6	0.600	0.200	0.625	0.125	0.500	-0.167
0.7	0.700	0.100	0.625	-0.125	0.667	-0.333
0.8	0.700	0.000	0.750	-0.125	0.667	-0.333
0.9	0.700	0.000	0.750	-0.125	0.667	-0.500
0.99	0.700	-0.100	0.750	-0.250	0.833	-0.500

Case 5: Income Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	0.583	0.5	0.333	0.167	0.333	0.167
0.5	0.583	0.333	0.500	0.000	0.500	0.000
0.6	0.583	0.333	0.500	0.000	0.500	0.000
0.7	0.667	0.167	0.667	-0.167	0.667	-0.167
0.8	0.667	0.167	0.667	-0.333	0.667	-0.333
0.9	0.667	0.167	0.667	-0.333	0.667	-0.333
0.99	0.667	0.000	0.833	-0.500	0.833	-0.500

Case 6: Income Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	0.562	0.500	0.250	0.125	a	a
0.5	0.562	0.375	0.500	0.125	a	a
0.6	0.562	0.375	0.500	0.125	a	a
0.7	0.562	0.313	0.625	0.125	a	a
0.8	0.562	0.313	0.625	0.125	a	a
0.9	0.562	0.313	0.625	0.125	a	a
0.99	0.562	0.313	0.625	0.125	a	a

Case 7: Income Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	0.500	0.500	0.000	0.000	0.000	0.000
0.5	0.500	0.500	0.000	0.000	0.000	0.000
0.6	0.500	0.500	0.000	0.000	0.000	0.000
0.7	0.500	0.500	0.000	0.000	0.000	0.000
0.8	0.500	0.500	0.000	0.000	0.000	0.000
0.9	0.500	0.500	0.000	0.000	0.000	0.000
0.99	0.500	0.500	0.000	0.000	0.000	0.000

Case 8: No Transfer Payments

	1		2		3	
δ	max	min	max	min	max	min
0.4	0.500	0.250	0.500	0.250	0.500	0.250
0.5	0.625	0.000	0.625	0.000	0.625	0.000
0.6	0.625	0.000	0.625	0.000	0.625	0.000
0.7	0.625	0.000	0.625	0.000	0.625	0.000
0.8	0.750	0.000	0.750	0.000	0.750	0.000
0.9	0.750	0.000	0.750	0.000	0.750	0.000
0.99	0.750	0.000	0.750	0.000	0.750	0.000

a - Tax rate is undefined because income and contributions are both zero.

b - G=1 is not sustainable.

Table V-b: Public Provision in a Dynamic Setting when G=1

Case 1: Identical Individuals

	1		2		3	
δ	max	min	max	min	max	min
0.4	1.000	-0.125	1.000	-0.125	1.000	-0.125
0.5	1.000	-0.250	1.000	-0.250	1.000	-0.250
0.6	1.000	-0.375	1.000	-0.375	1.000	-0.375
0.7	1.000	-0.500	1.000	-0.500	1.000	-0.500
0.8	1.000	-0.625	1.000	-0.625	1.000	-0.625
0.9	1.000	-0.625	1.000	-0.625	1.000	-0.625
0.99	1.000	-0.625	1.000	-0.625	1.000	-0.625

Case 2: Taste Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	1.000	-0.125	1.000	-0.125	1.000	-0.125
0.5	1.000	-0.250	1.000	-0.375	1.000	-0.375
0.6	1.000	-0.375	1.000	-0.500	1.000	-0.500
0.7	1.000	-0.500	1.000	-0.625	1.000	-0.625
0.8	1.000	-0.625	1.000	-0.625	1.000	-0.625
0.9	1.000	-0.625	1.000	-0.625	1.000	-0.625
0.99	1.000	-0.625	1.000	-0.625	1.000	-0.625

Case 3: Taste Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	0.875	0.250	1.000	-0.250	1.000	-0.250
0.5	1.000	0.000	1.000	-0.500	1.000	-0.500
0.6	1.000	-0.125	1.000	-0.625	1.000	-0.625
0.7	1.000	-0.375	1.000	-0.625	1.000	-0.625
0.8	1.000	-0.500	1.000	-0.625	1.000	-0.625
0.9	1.000	-0.625	1.000	-0.625	1.000	-0.625
0.99	1.000	-0.625	1.000	-0.625	1.000	-0.625

Case 4: Income Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	1.000	0.100	1.000	-0.125	1.000	-0.500
0.5	1.000	0.000	1.000	-0.250	1.000	-0.667
0.6	1.000	-0.100	1.000	-0.375	1.000	-0.833
0.7	1.000	-0.200	1.000	-0.500	1.000	-1.000
0.8	1.000	-0.300	1.000	-0.625	1.000	-1.167
0.9	1.000	-0.300	1.000	-0.625	1.000	-1.167
0.99	1.000	-0.300	1.000	-0.625	1.000	-1.167

Case 5: Income Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	1.000	0.250	1.000	-0.500	1.000	-0.500
0.5	1.000	0.167	1.000	-0.667	1.000	-0.667
0.6	1.000	0.083	1.000	-0.833	1.000	-0.833
0.7	1.000	0.000	1.000	-1.000	1.000	-1.000
0.8	1.000	-0.083	1.000	-1.167	1.000	-1.167
0.9	1.000	-0.083	1.000	-1.167	1.000	-1.167
0.99	1.000	-0.083	1.000	-1.167	1.000	-1.167

Case 6: Income Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	1.000	0.438	1.000	-0.125	a	a
0.5	1.000	0.375	1.000	-0.250	a	a
0.6	1.000	0.312	1.000	-0.375	a	a
0.7	1.000	0.250	1.000	-0.500	a	a
0.8	1.000	0.187	1.000	-0.625	a	a
0.9	1.000	0.187	1.000	-0.625	a	a
0.99	1.000	0.187	1.000	-0.625	0.000	0.000

Case 7: Income Heterogeneity

	1		2		3	
δ	max	min	max	min	max	min
0.4	1.000	0.550	1.000	-3.500	1.000	-3.500
0.5	1.000	0.500	1.000	-4.000	1.000	-4.000
0.6	1.000	0.450	1.000	-4.500	1.000	-4.500
0.7	1.000	0.400	1.000	-5.000	1.000	-5.000
0.8	1.000	0.350	1.000	-5.500	1.000	-5.500
0.9	1.000	0.350	1.000	-5.500	1.000	-5.500
0.99	1.000	0.350	1.000	-5.500	1.000	-5.500

Case 8: No Transfer Payments

	1		2		3	
δ	max	min	max	min	max	min
0.4	1.000	0.000	1.000	0.000	1.000	0.000
0.5	1.000	0.000	1.000	0.000	1.000	0.000
0.6	1.000	0.000	1.000	0.000	1.000	0.000
0.7	1.000	0.000	1.000	0.000	1.000	0.000
0.8	1.000	0.000	1.000	0.000	1.000	0.000
0.9	1.000	0.000	1.000	0.000	1.000	0.000
0.99	1.000	0.000	1.000	0.000	1.000	0.000

a - Tax rate is undefined because income and contributions are both zero.
b - G=1 is not sustainable.

Figure 1: Sustainable Outcomes with Identical Individuals (Case 1)

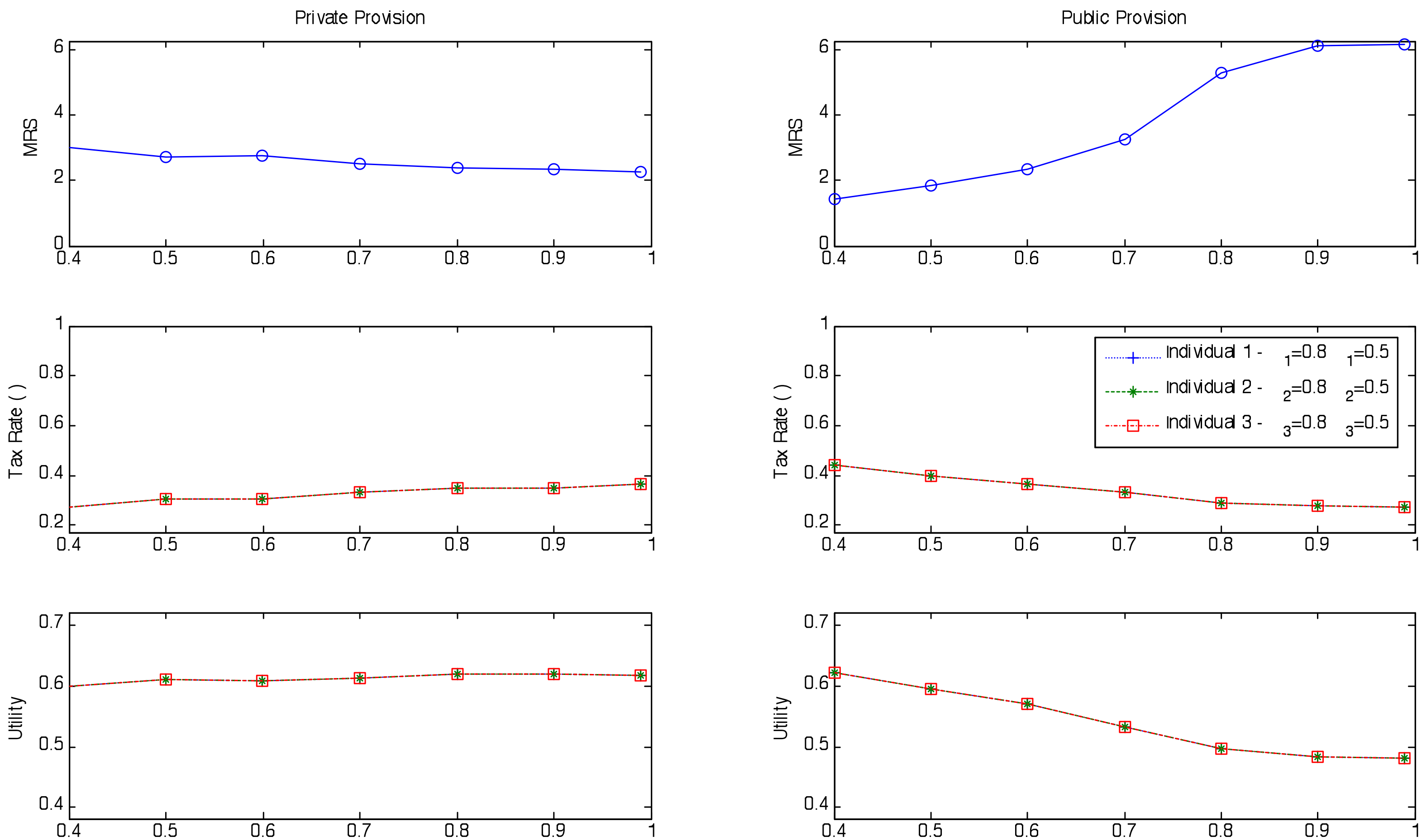

Figure 2: Sustainable Outcomes with Taste Heterogeneity (Case 2)

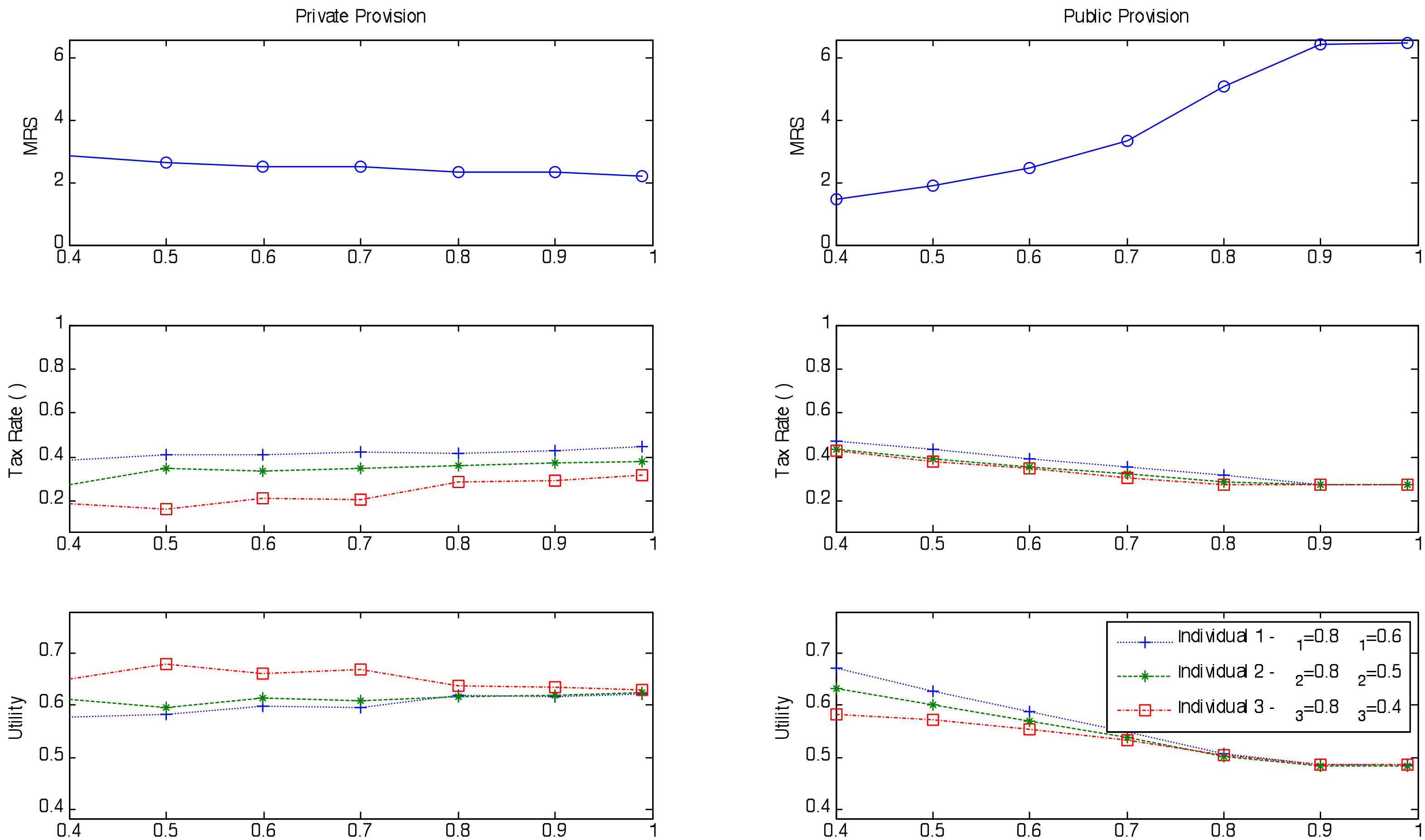

Figure 3: Sustainable Outcomes with Taste Heterogeneity (Case 3)

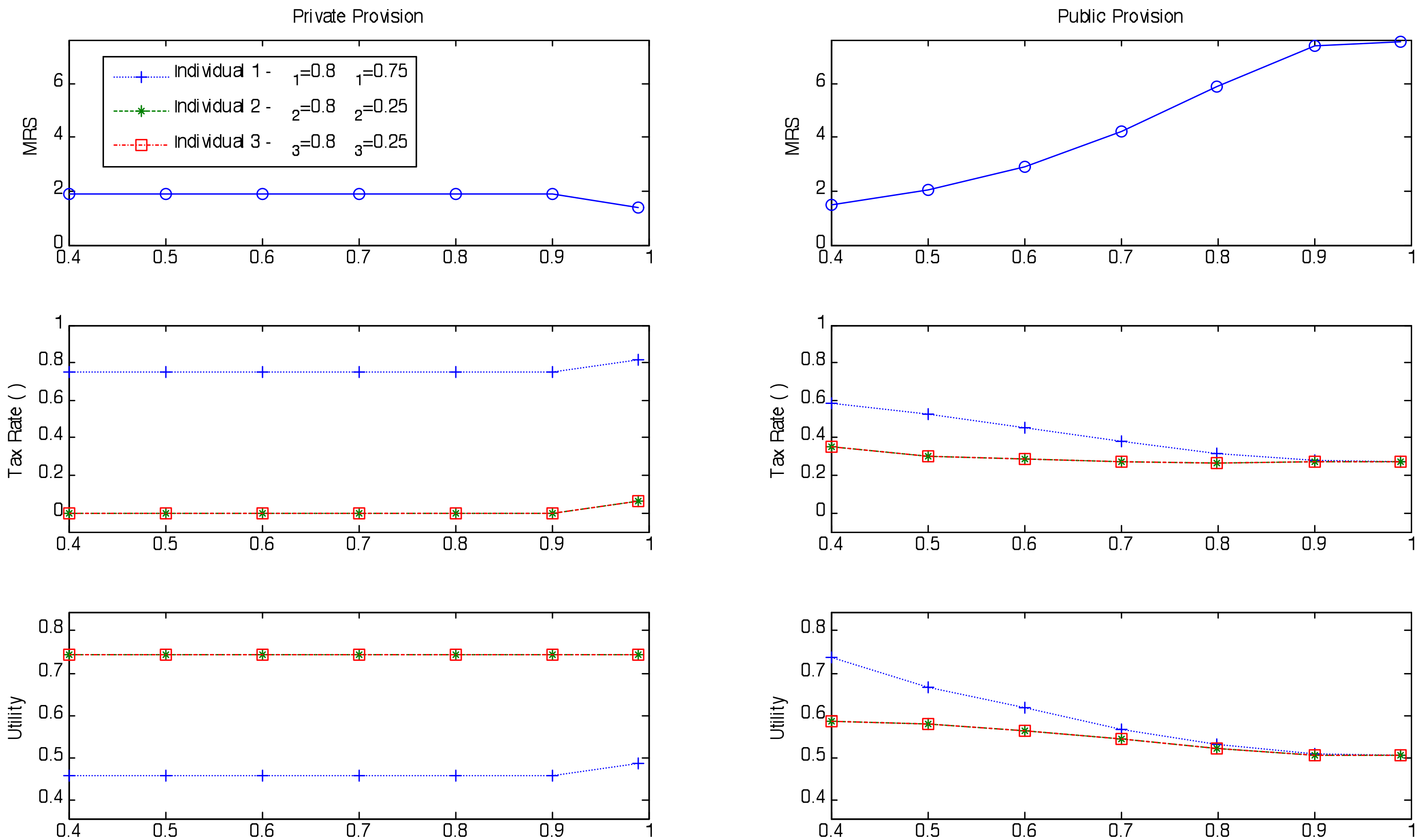

Figure 4: Sustainable Outcomes with Income Heterogeneity (Case 4)

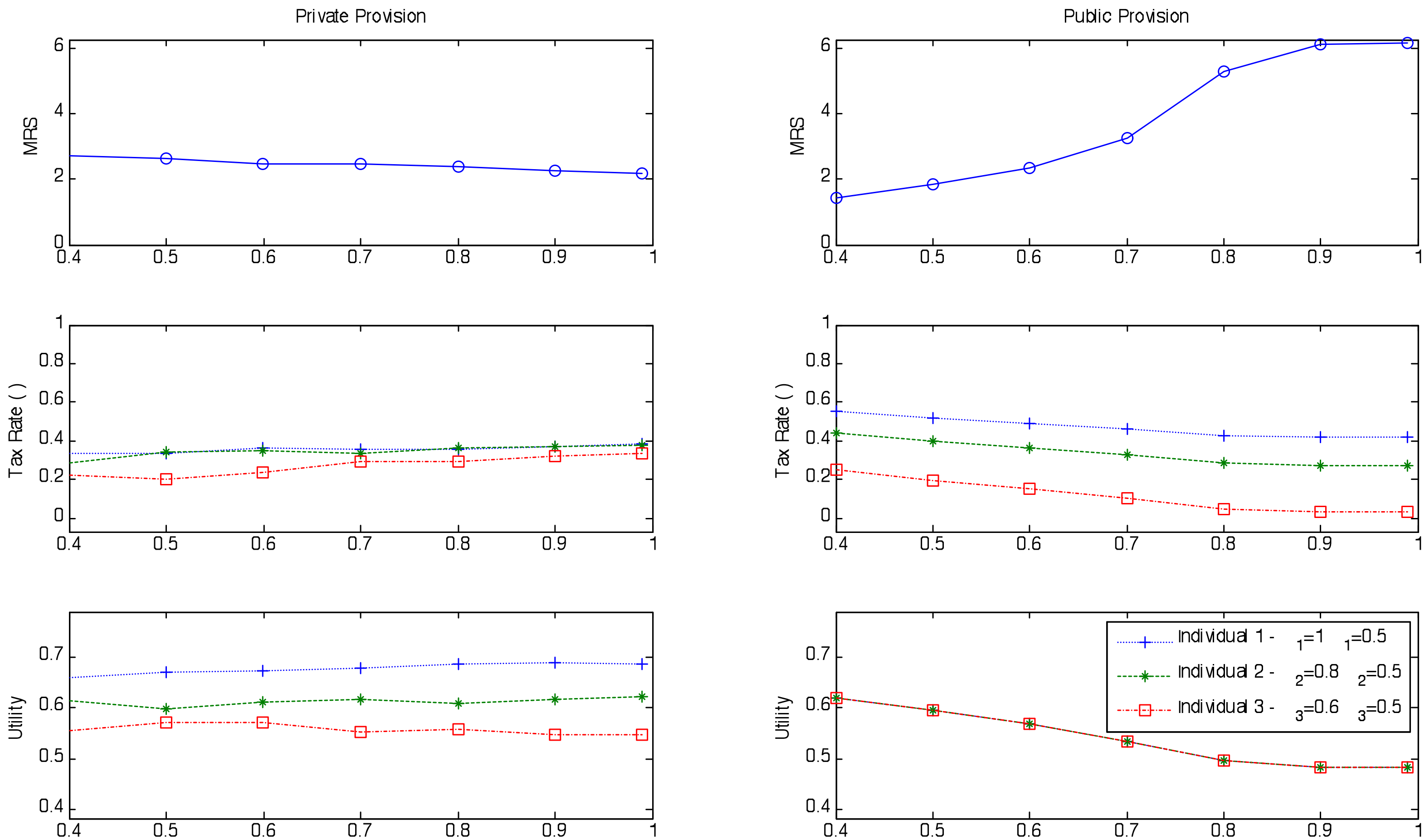

Figure 5: Sustainable Outcomes with Income Heterogeneity (Case 5)

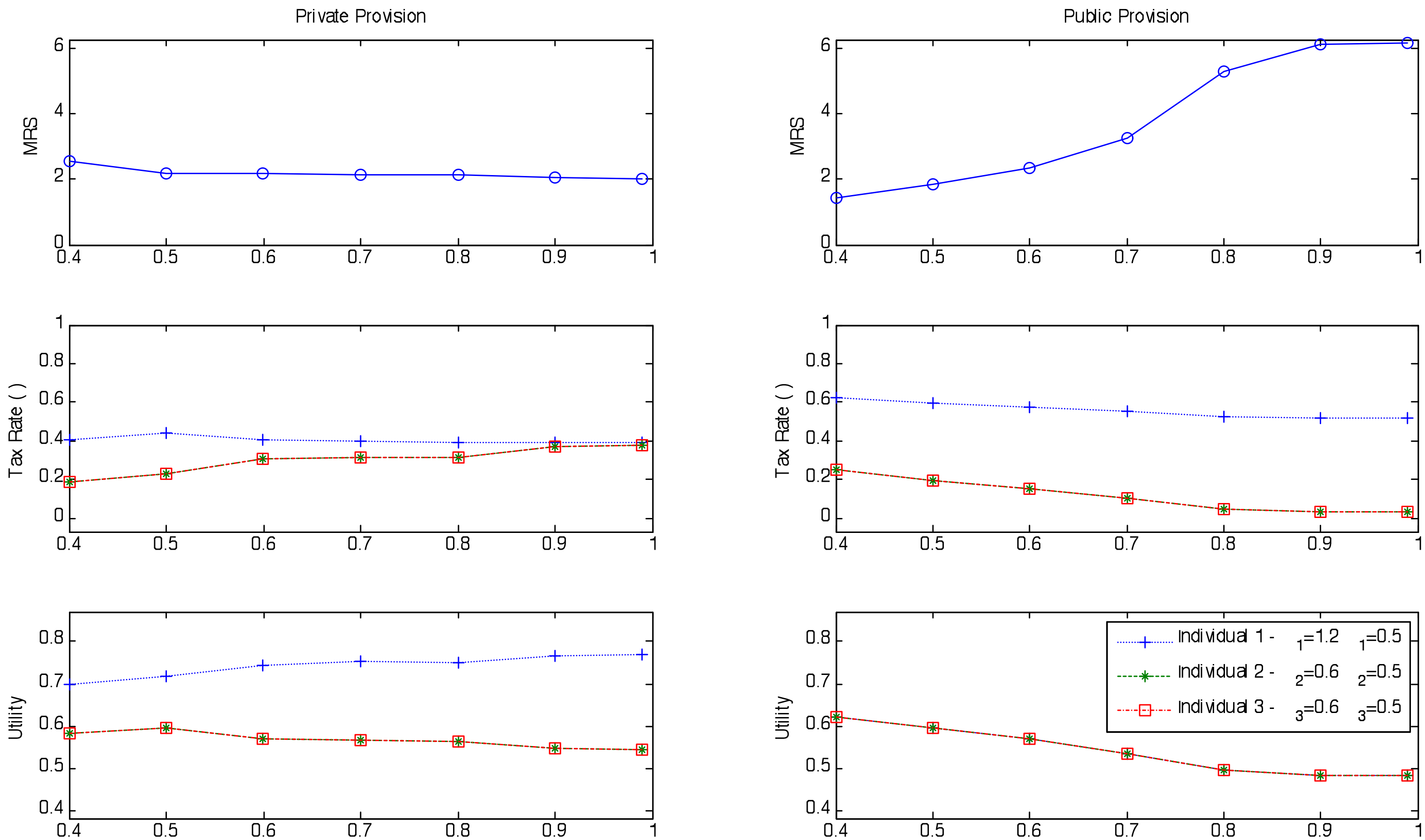

Figure 6: Sustainable Outcomes with Income Heterogeneity (Case 6)

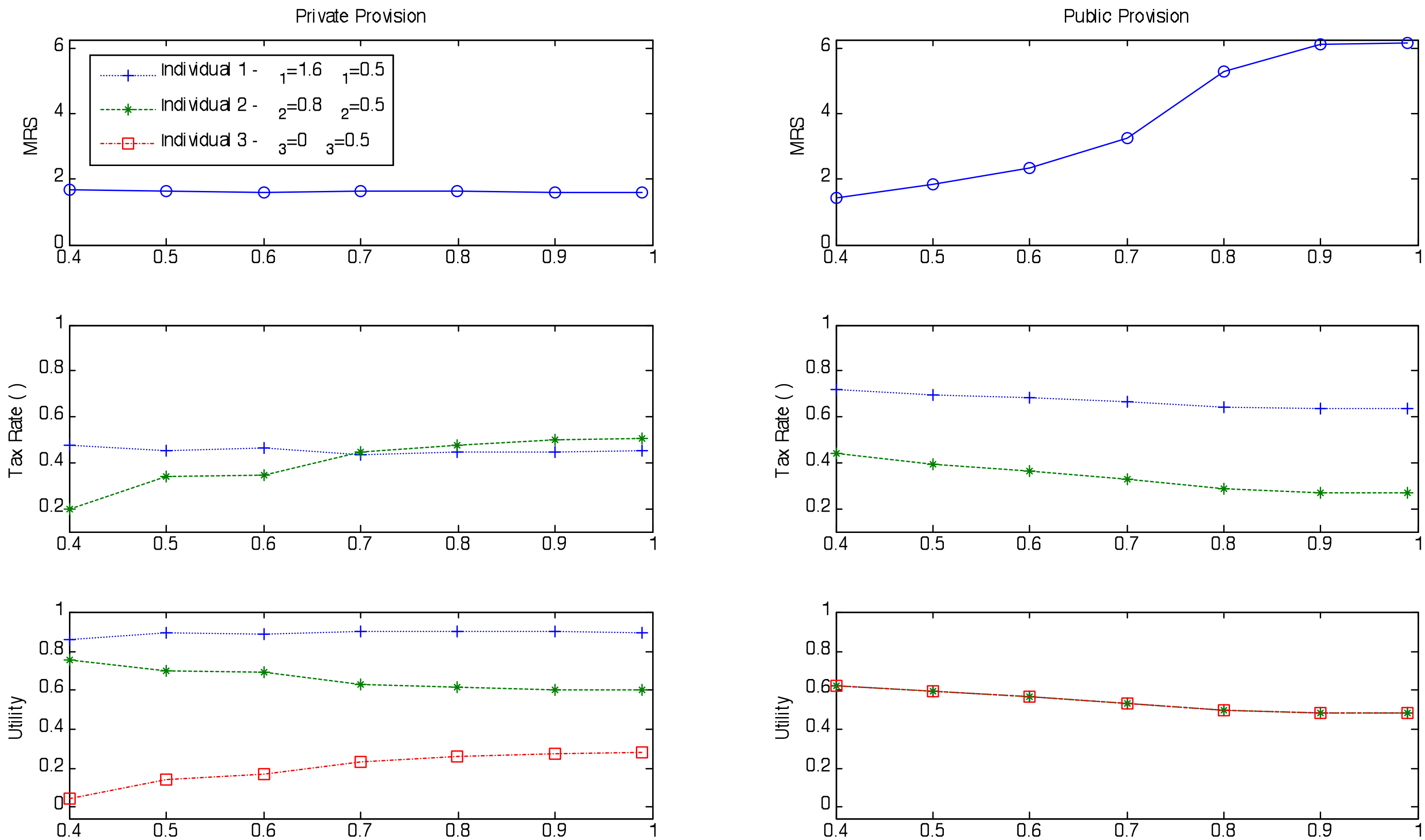

Figure 7: Sustainable Outcomes with Income Heterogeneity (Case 7)

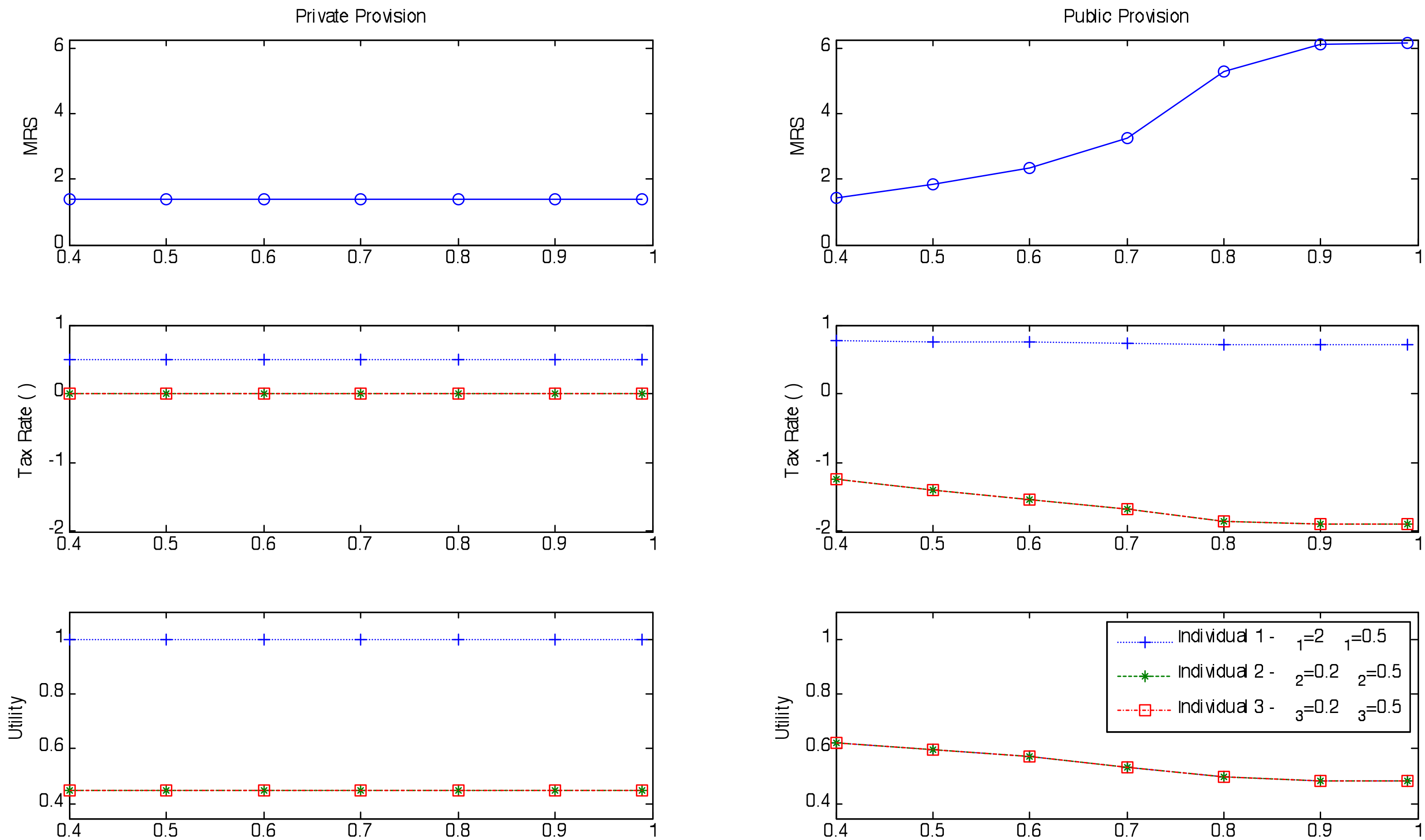

Figure 8: Sustainable Outcomes with Identical Individuals and No Transfer Payments (Case 8)